We stir wind–tunnel turbulence with an active grid that consists of rods with attached vanes. The time–varying angle of these rods is controlled by random numbers. We study the response of turbulence on the statistical properties of these random numbers. The random numbers are generated by the Gledzer–Ohkitani–Yamada shell model, which is a simple dynamical model of turbulence that produces a velocity field displaying inertial–range scaling behavior. The range of scales can be adjusted by selection of shells. We find that the largest energy input and the smallest anisotropy are reached when the time scale of the random numbers matches that of the largest eddies of the wind–tunnel turbulence. A large mismatch of these times creates a flow with interesting statistics, but it is not turbulence.

The standard way to stir turbulence in a wind tunnel is by passing a laminar wind through a grid that consists of a regular mesh of bars or rods. In this way, near–homogeneous and near–isotropic turbulence can be made with the largest eddy size set by the mesh size, but the maximum attainable turbulent Reynolds number is small [1]. The precise structure of such a grid should not matter much; according to Kolmogorov’s theory, any stirred turbulent flow, if left to its own devices, becomes homogeneous and isotropic at small enough scales [2].

However, in practical situations which involve finite Reynolds numbers, these scales may not be reached before viscosity starts to act. Therefore, the manner in which turbulence is stirred matters in practice, and grids which have a multiscale structure have recently been found to produce interesting flow statistics [3].

Much more vigorous turbulence can be stirred by so–called active grids that have moving elements. In this way, Taylor–based Reynolds numbers $Re_\lambda \approx 10^3$ can be achieved in flows that are homogeneous and near–isotropic. To stir turbulence, the space–time structure of active grids is controlled by random numbers. This Letter addresses the question how these random numbers should be chosen to create the desired turbulence statistics. Rather than using a (pseudo-) random number generator, we will generate these random numbers with the help of a simple shell model of turbulence. These random numbers have spectral distributions that are similar to those of turbulence, while the statistical properties of these numbers can be quantified with turbulence parameters, such as integral length- and time scales. Thus we are stirring turbulence with turbulence.

Active grids, such as the one used in our experiment, were pioneered by Makita [4] and consist of a grid of rods with attached vanes that can be rotated by servo motors. Several simple protocols, involving uncorrelated uniformly distributed random numbers to rotate the axes have been tried [5, 6].

In this Letter we study the relation between the statistical properties of a model–turbulence signal that drives the grid and those of the wind–tunnel turbulence. We will find that isotropic turbulence results when matching the integral time scales, but also that wind–tunnel turbulence with unusual properties, such as highly intermittent large–scale statistics, can emerge from a gross mismatch of time scales. This is remarkable because intermittency, the non–Gaussian statistics of the velocity field, is a property of turbulent velocity differences measured over length scales much smaller than the stirring scale. This may find an application in wind–tunnel tests that seek to model the intermittent atmosphere.

A schematic drawing of the wind tunnel and a picture of the active grid is shown in Fig. 1. The active grid is placed in the 8 m long experimental section of a recirculating wind tunnel. Turbulent velocity fluctuations are measured at a distance $4.6$ m downstream from the grid using a single x–wire anemometer. Our grid has mesh size $M = 0.1$ m and consists of 17 axes whose instantaneous angles $\theta_i(t), i = 1, \ldots, 17$ are prescribed precisely using PID controllers. Angle information can be fed to the grid with a sampling time $\delta t_g = 10^{-2}$ s. From an
experiment where we drive the grid with white noise, we estimate that the response time of the grid is \( \approx 4 \times 10^{-3} \) s. This response time is limited by the inertia of the axes; it also limits the fastest turbulence time scale that can be directly imposed on the flow through the active grid. In a typical experiment we feed a time series computed by the turbulence model to the grid and collect wind data during 450 s \( (4.5 \times 10^3 \text{ large–eddy turnover times}) \).

We measure the \( u, w \) velocity components of the flow, which allows an assessment of the flow isotropy. The locally manufactured hot–wire velocity probe had a 2.5 \( \mu \text{m} \) diameter and a sensitivity of 400 \( \mu \text{m} \), which is comparable to the typical smallest length scale of the flow in our experiments (the measured Kolmogorov scale is \( \eta \approx 170 \mu \text{m} \)). The hot–wire probe was operated at constant temperature using computer controlled anemometers that were also developed locally. Each experiment was preceded by a calibration procedure \([7, 8]\). The signals captured by the sensors were sampled at 20 kHz, after being low-pass filtered at 10 kHz.

The turbulence model used for driving the grid is the Gledzer–Ohkitani–Yamada (GOY) shell model which is a dynamical model for the time dependent amplitudes \( u_n(t) \) of Fourier modes at wavenumbers \( k_n \) \([9–11]\),

\[
\left( \frac{d}{dt} + \nu k_n^2 \right) u_n = i \left( a_n u_{n+1} u_{n+2} + b_n u_{n-1} u_{n+1} \right) + c_n u_{n-1} u_{n-2} + f_n,
\]

where \( \nu = 10^{-7} \) is the kinematic viscosity, \( k_n \) is an exponentially spaced grid of scalar wave numbers, \( k_n = 2^{n-4}, n = 1, \ldots, 22 \), and where the factors \( a_n = k_n, b_n = -k_{n-1}/2, c_n = -k_{n-2}/2 \) ensure conservation of energy, enstrophy and helicity in the inviscid \( (\nu = 0) \) and unforced system. The model mimics the Navier–Stokes equation in wavenumber space, but as the wavenumbers are scalar, it lacks the flow structures typical for turbulence, such as vortices. Further, the nonlinear advection term which in the Navier–Stokes equation couples all wavenumbers to all others, is here restricted to interactions between neighboring wavenumbers and the time dependence of the velocity field is that of a Lagrangian one \([12]\). In our simulations we followed the standard numerical approach of \([13]\) and forced the first shell, \( f_n = 5 \times 10^{-3} \delta_{n,1} (1 + t) \), with \( t \) the imaginary unit.

In our experiment we modulate turbulence by passing a laminar wind with velocity \( U \approx 10 \text{ ms}^{-1} \) through the active grid; as the mean velocity is much larger than the fluctuating velocity, we supply Eulerian velocity fields, \( u(x, t) \), with \( x = U t \). Strictly, therefore, we stir a Eulerian field with Lagrangian velocities.

Let us now summarize the statistical properties of the shell–model velocity field. From the complex shell velocities we compute the space–time turbulent velocity field \( u(x, t) = \Re \sum_n e^{ik_n x} u_n(t) \). The energy dissipation \( \epsilon = 3.7 \times 10^{-3} \), which is computed from the energy input in the forced shell, \( \epsilon = \Re (u^*_x f_x) \), defines the Kolmogorov length– \( \eta \approx (\nu^3/\epsilon)^{1/4} = 2.3 \times 10^{-5} \), and time scales \( \tau_\eta \approx (\nu/\epsilon)^{1/2} = 5.2 \times 10^{-3} \), and the Taylor Reynolds number \( \text{Re}_\Lambda = u^2 (\nu^3/\epsilon)^{-1/2} = 5.3 \times 10^3 \), with \( u \) the turbulent velocity, \( u^2 = \langle u^2(0, t) - \langle u(0, t) \rangle^2 \rangle \). The integral length– and time scales are computed from the wavenumber and frequency spectra, \( L = \pi E(k_1)/2 u^2 = 7.5 \), with \( E(k_n) = k_n^{5/2} \langle |u_n|^2 \rangle \), and \( T = \pi E(f = 0)/2 u^2 = 46 \), respectively.

To allow for its finite response time, we drive the grid with a low–pass filtered velocity field restricted to the first \( N \) shells \([14]\),

\[
u^{(N)}(x, t) = \Re \sum_n e^{ik_n x} u_n(t),
\]

with the associated frequency spectra \( E_N(f) = \langle |f u^{(N)}(x = 0, t) e^{-2\pi i f t} dt|^2 \rangle \). The associated cut–off frequency \( f_N \) scales with the cut–off wave number \( k_N \) as \( f_N \propto k_N^{2/3} \propto 2^{2N/3} \).

Time traces of shell–model velocities, with both the velocity field restricted to \( N = 2 \) and the fully resolved velocity field \( (N = 22) \) are shown in Fig. 2. As illustrated by the spectra in Fig. 3(a), the frequency content of these two signals differs by two orders of magnitude. The fully resolved velocity is characterized by long episodes of relatively calm, interspersed with short bursts of turbulent activity. This highly intermittent character is in accordance with the very large Reynolds number.

The statistical properties of the simulated turbulent velocity are illustrated in Fig. 3. The frequency spectrum \( E_{22}(f) \) shows the expected inertial–range behavior \( E(f) \sim f^{-2} \), while the cut–off frequencies \( f_N \) of the filtered spectra \( E_N(f) \) scale as \( f_N \propto 2^{2N/3} \). The probability density function (PDF) \( P(\Delta u) \) of temporal velocity increments evolves from Gaussian at large
time delays, to highly intermittent stretched exponential \( P(\Delta u) \sim \exp(-\beta|\Delta u|^{\alpha}) \), \( \alpha < 1 \) for the shortest time delays.

The GOY model produces a space–time signal \( u^{(N)}(x, t) \) [Eq. (2)] which we translate to axis–angle signals \( \theta_i(t) \) to drive the grid. There is not a simple relation between \( \theta_i(t) \) and the flow velocity. It appears that only at very low grid frequencies the mean flow approximately follows the time–varying transparency of the grid [15]. We therefore take the simple approach to set the angle axes \( \theta_i(t) \) proportional to the fluctuating velocity \( u(x_i, t) \) of the GOY model, where discrete \( x_i = iM \) refer to the axes of the grid.

The quantities of the shell model are dimensionless and must be expressed in physical units. To this aim we fix the reference time scale \( T^* \) of the experiment to the value \( T^* = 0.1 \) s which is typical for our conditions and introduce a velocity field in physical units \( u(x, t) = C_u \Re \sum_i e^{ik_i x} C_i(t) \), choosing \( C_u = 2\pi/3u \). The smallest length scale of the driving signal \( u^{(N)}(x, t) \) is \( 2\pi/k_N \), which we associate with the grid mesh size by choosing \( C_x \) such that \( C_x k_N M = 2\pi \). The grid moves spatially incoherently at this scale, but is correlated at larger scales \( > 2\pi/k_N \). The time scale \( C_t = T^*/T^g \), is chosen such that at \( T^g = T \) we drive the grid with the integral time of the wind–tunnel turbulence. Therefore, the ratio \( T/T^g \) is a measure of the mismatch between the integral time of the wind–tunnel turbulence and that of the random signal driving it.

At \( N = 2 \), with the simulated velocity signal limited to the first two shells, the fastest time scale in the signal is \( 39 \tau_u \), while the ratio of the integral time scales is \( T/T^g = 23 \). For the integral time scales of the driving signal to match that of the turbulence, this signal must be sped up. Since the grid sampling time is limited to \( \delta_{ts} = 10^{-2} \) s, this is done by low-pass filtering and sub–sampling the driving signal. For increasing \( N \), the driving signal contains more and more small–scale fluctuations. At \( N = 10 \), the smallest time step supplied is approximately the Kolmogorov time, and an extreme mismatch between the integral scales of the driving and that of the wind–tunnel turbulence results. It is as if we try to impose the intermittent viscous scale statistics of the GOY model at integral scales of the experiment.

In our experiment we vary the relative time scale \( T/T^g \), and measure the properties of the turbulence generated. We characterize the generated wind–tunnel turbulence by its anisotropy and the dissipation rate \( \epsilon \). The (pseudo-) energy dissipation rate \( \epsilon \) was inferred from a single derivative, \( \epsilon = 15\nu(\partial u/\partial x)^2 \), with \( \nu \) the kinematic viscosity, but the one involving \( \partial w/\partial x \) showed a similar dependence on \( T/T^g \).

The dependence of the dissipation rate \( \epsilon \) on the relative time scale \( T/T^g \) is shown in Fig. 4(a). Clearly, the optimum energy input into turbulence is reached when the time scale of the random time series from the GOY model matches the experimental integral time, \( T/T^g = 1 \).

The maximum of the energy dissipation as a function of the relative times \( T/T^g \), is largest for a cut–off shell index \( N = 4 \). This behavior can be understood from the spectral energy cascade. At \( N = 4 \), grid cells are incoherent on a scale \( k_4 \), which implies approximate coherence on a scale \( k_2 \) as the energy in this scale is a factor \( 4^{5/3} \approx 10 \) larger than that in scale \( k_4 \). This implies that two halves in one direction of the grid are moving incoherently, resulting in shear layers that have the extent of a few times the integral length scale and a large energy input. At smaller \( N \) these shear layers are too small, while at larger \( N \), the entire grid moves coherently, with small variations at the higher shell numbers. No shear layers result and now the grid is choking the entire wind–tunnel flow at random times.

Driving the grid with matched time scales, \( T/T^g \approx 1 \) also results in turbulence with the smallest anisotropy as shown in Fig. 4(b). At a large mismatch, also the anisotropy becomes large. Similar behavior is found for the small–scale anisotropy inferred from the energy spec-
extra of the two velocity components.

A question is how the statistics of the stirred turbulence is affected by a large mismatch with the stirring scale, in particular if we could endow large–scale turbulence with small–scale statistics. The remarkable answer to this question is illustrated in Fig. 5(a,b) which show probability density functions of longitudinal velocity increments at integral–scale separation, \( r/\eta = 1200 \) and dissipative scale \( r/\eta = 8 \). The dashed lines are Gaussians \( P(x) = \pi^{1/2} \exp(-x^2) \) with \( x = \Delta u/\Delta u_{\text{rms}} \). (a) For time series I, at \( T/T^\ast = 1.4 \) (b) for time series II at \( T/T^\ast = 45 \). (c) Third–order structure function for the case I, II. For case I it is compared to the Kolmogorov prediction \( G_3 = -(4/5)\epsilon r \) (dashed line).

We conclude that in stirring turbulence with an active grid, not just the distribution of the random numbers that drive the grid matters, but also their correlation properties. The GOY model allowed us to produce random numbers with properties that can be expressed in terms of turbulence quantities, such as the integral and dissipative time scales. Turbulence needs to be stirred with random numbers whose integral time scale matches that of the wind–tunnel flow. A large mismatch leads to interesting statistics of the velocity increments in the windtunnel, but the small–scale velocity statistics no longer satisfy the fundamental Kolmogorov relation: turbulence can not be fooled.

Being able to specify the correlation properties of the random signal driving the grid, is an essential refinement of the control of active–grid motion. The number of independent grid cells in our grid limits the dynamic range of spatial scales, however, active grids are now underway that have a much larger number of cells.

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FIG. 5: (a, b) Full lines show probability density functions of longitudinal velocity increments at integral scale, \( r/\eta = 1200 \) and dissipative scale \( r/\eta = 8 \). The dashed lines are Gaussians \( P(x) = \pi^{1/2} \exp(-x^2) \) with \( x = \Delta u/\Delta u_{\text{rms}} \). (a) For time series I, at \( T/T^\ast = 1.4 \) (b) for time series II at \( T/T^\ast = 45 \). (c) Third–order structure function for the case I, II. For case I it is compared to the Kolmogorov prediction \( G_3 = -(4/5)\epsilon r \) (dashed line).