Frequency domain estimation of parabolic partial differential equations with spatially varying transport coefficients

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Introduction

Many distributed systems are described by a parabolic partial differential equation known as the heat equation. It describes how heat is transported throughout media such as air, fluids, plasma, and solids. However, to arrive at accurate descriptions necessary for control of such systems its unknown profiles, transport coefficients as function of space, needs to be identified.

A new one-dimensional estimation code is under development which allows the identification of such profiles using a distributed least-squares cost function in the frequency domain. The code conceptionally works, but needs to be made robust for various profiles and different boundary conditions. Hence, the objective of this project is to further develop the code such that it can be used in practical applications. It is foreseen that this code is going to be used in the near future in three practical applications: 1) to estimate the heat source and transport coefficients of the electron heat transport in fusion reactors [1]; 2) the heat transport in materials such as soil and ink; 3) and contaminant transport in ground water studies [2]. These are also visually depicted in Fig. 1.

![Joint European Torus largest tokamak fusion reactor](image1.png)  ![Measurements of temperature fluctuations in the river bed](image2.png)

Fig. 1: Joint European Torus largest tokamak fusion reactor (left). Measurements of temperature fluctuations in the river bed to study spreading contaminants in the groundwater.

Problem statement

In this project, the goal of the student is to:

1. Obtain in-depth knowledge of the existing frequency domain estimation algorithms for parabolic PDEs.
2. Implement validation tests such as cost-function and whiteness residual tests.
3. Implement and test various spatial basis-functions on simulation models for the profiles to be identified.
4. Investigate various possibilities to reduce numerical errors and numerical instabilities.
5. Extend to cylindrical domains and possibly to two-dimensions.

Requirements

Basic knowledge on system theory and system identification. Type of work: 25% theory, 75% application (with a large programming components).

References:
