Data-Based Modeling of Continuous-Time Lur'e-Type Systems with Stability Certification

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SISO Lur'e-Type Systems
- LTI Dynamics
  \[ \dot{x} = Ax + Bu + Lw \]
  \[ y = Cx + Dw \]
  \[ z = Fx + Gu + Hw \]
  - Static Nonlinearity: \( u = \phi(y) \)
- Convergent Dynamics
  \[ \dot{x} = F(x(t), w(t)) \]
  - Unique Globally Exponentially Stable Steady-State \( \bar{x}(t) \)
  - Verifiable Conditions for Lur'e-Type Systems

Modeling Problem
- Periodic Input
- True System Assumed Convergent
- Steady-State Mismatch
- Lur'e-Type System (\( \theta \)) Parametrization Given
- Constrained Optimization Problem
  \[ \hat{\theta} = \arg \min_{\theta} \frac{1}{N} \sum_{k=1}^{N} (z_{\text{simulated}}(t_k, \theta) - z_{\text{measured}}(t_k))^2 \]

Two-Step Optimization Strategy
- Step 1: Initialization
  - Global Optimization
  - Linear Approximation
  - Physical Insights
- Step 2: Constrained Gradient-Based Search

Fast Mixed-Time-Frequency Algorithm
- Frequency Domain
- LTI Dynamics
- FFT
- Contraction Mapping
- Static Nonlinearity
- IFFT

Exact Gradient Computation
- Input \( w \)
- Lur'e-Type System
- Sensitivity
- Fast & Exact Computation:
  - Steady-State Model Response \( \rightarrow \) Evaluate Cost Function
  - Steady-State Model Sensitivity \( \rightarrow \) Compute Gradient of Cost Function

Mechanical Ventilation Setup
- "Patient"
- Hose
- Blower
- Output Airway Pressure
- Variety of Patient-Hose Configurations
- Restricted Shape of Input Signal

Results
- Accurate & Convergent Model
- Fast: 1530 Model Responses Computed in 14 Seconds
- Measured Response
- Airway Pressure [mba]
- Time [sec]
- Initial Cost: 2.55
- Final Cost: 2.40
- Model Cost: 0.17

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